ANSWER KEY

1.	C
2.	D
3.	A
4.	B
5.	A
6.	D
7.	C
8.	B
9.	B
10.	A
11.	C
12.	A
13.	B
14.	B
15.	A
16.	B
17.	B
18.	C
19.	D
20.	C
21.	D
22.	A
23.	B
24.	A
25.	A
26.	B
27.	C
28.	C
29.	A
30.	B

SOLUTION

1. $(1-p)$ is a root of
$x^2 + px + (1-p) = 0$.
$\therefore (1-p)^2 + p(1-p) + (1-p) = 0$
$(1-p)[1-p+p+1] = 0$
$\therefore \quad (1-p).\, 2 = 0 \Rightarrow \boxed{p=1}$
$\therefore \quad$ Equation is $x^2 + px + (1-p) = 0$
$\Rightarrow \quad x^2 + x = 0$

$x = 0 \qquad x = -1$

2. $\sqrt{2x+1} - \sqrt{2x-1} = 1 \qquad \left(x \geq \frac{1}{2}\right)$

Domain $x \geq \frac{1}{2} \qquad \Rightarrow \sqrt{4x^2-1} = ?$
$\sqrt{2x+1} = 1 + \sqrt{2x-1}$
$\Rightarrow \quad 2x + 1 = 1 + 2x - 1 + 2\sqrt{2x-1}$
$\Rightarrow \quad 1 = 2\sqrt{2x-1}$
Square $1 = 4(2x-1)$
$\Rightarrow \quad 1 = 8x - 4$
$8x = 5$, So

$x = \frac{5}{8}$

Then $\Rightarrow \sqrt{4x^2-1}$
$= \sqrt{4.\frac{25}{64}-1}$
$= \frac{3}{4}$

3. No solution

4. Given
$(a^2+b^2)x^2 - 2(bc+ad)x + (c^2+d^2) = 0$
$\Rightarrow \quad (ax-d)^2 + (bx-c)^2 = 0$
Which is possible, only when
$ax - d = 0 \quad \& \quad bx - c = 0$
$\therefore \ x = \frac{d}{a} \qquad \therefore \ x = \frac{c}{b}$

5. Given, $(a+b-c)x^2 - 2ax + (a-b+c) = 0$
Here $D = 4a^2 - 4(a-b+c).(a+b-c)$
$\therefore \quad D = 4\left[a^2 - [a+(b-c)].[a-(b-c)]\right]$
$= 4\left[a^2 - [a^2-(b-c)^2]\right]$
$= 4(b-c)^2$
$\therefore \quad$ D is positive & perfect square
$\therefore \quad$ Roots are rational

6. Given roots of equation :
$ax^2 + x + b = 0$ are real & unequal
$\therefore \ D > 0$
$\boxed{1-4ab>0}$
Now the roots of
$x^2 - 4\sqrt{ab}\,x + 1 = 0$
Here $D = 16ab - 4 = 4(4ab-1)$
$\therefore \quad D < 0$
$\therefore \quad$ Roots and complex

7. $\because\ 3\alpha + 4\beta = 7$

$\&\ 5\alpha - \beta = 4$

$\therefore\ \alpha = \beta = 1$

Now $x^2 + px + q = 0$

Here $\alpha + \beta = -p \Rightarrow p = -2$

$\&\ \alpha\beta = q \Rightarrow q = 1$

8. One root of $ax^2 + bx + c = 0$ is square of other root if

$ac^2 + a^2c + b^3 = 3abc$

Here $a = 1; b = -30; c = p$

$p^2 + p - 27000 = -90p$

$\therefore\ p^2 + 91p - 27000 = 0$

$\therefore\ p = 125, -216$

9. $\frac{a}{x-a} + \frac{b}{x-b} = 1$

$\Rightarrow a(x-b) + b(x-a) = (x-a)(x-b)$

$\Rightarrow ax - ab + bx - ab = x^2 - (a+b)x + ab$

$\Rightarrow x^2 - 2(a+b)x - 3ab = 0$

Now $\alpha + \beta = 0$

$\therefore\ a + b = 0$

10. $(\alpha - \beta) < \sqrt{5}$

$(\alpha - \beta)^2 < 5$

$(\alpha + \beta)^2 - 4\alpha\beta < 5$

$\Rightarrow a^2 - 4 < 5$

$\Rightarrow a^2 < 9$

$\Rightarrow |a| < 3$

$\therefore\ a \in (-3, 3)$

11. Here roots of $x^2 - x + 1 = 0$ are $-\omega$ and $-\omega^2$

$\Rightarrow (-\omega)^{2009} + (-\omega^2)^{2009}$

$= (-\omega)^{2007}(-\omega)^2 + (-\omega^2)^{2007}(-\omega^2)^2 = -(\omega^2 + \omega)$

$= 1$

12. $\alpha + \beta = -\frac{q}{p};\ \alpha\beta = \frac{r}{p}$

$2q = p + r$

$\frac{\alpha+\beta}{\alpha\beta} = 4 \Rightarrow \text{given}$

$\frac{-q}{r} = 4 \Rightarrow 2(-4r) = p + r \Rightarrow p = -9r$

$|\alpha - \beta| = \frac{\sqrt{D}}{|p|} = \frac{\sqrt{q^2 - 4pr}}{|p|} = \frac{\sqrt{16r^2 - 4pr}}{|p|}$

$= \frac{\sqrt{52r^2}}{|p|} = \frac{2|r|\sqrt{13}}{9|r|} = \frac{2\sqrt{13}}{9}$

13. It is $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$.

14. $\alpha = \frac{1}{1+i} = \frac{1-i}{2}$

$\therefore\ \beta = \frac{1+i}{2}$

Also $S = \alpha + \beta = 1$

$\&\ P = \alpha.\beta = \frac{1+1}{4} = \frac{1}{2}$

$\therefore$ equation is $x^2 - Sx + P = 0$

$x^2 - x + \frac{1}{2} = 0$

$\Rightarrow 2x^2 - 2x + 1 = 0$

15. $x^2 - 2x + 3 = 0 \rightarrow$ Roots are α, β

To find the equation whose roots are $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

$\because\ X = \frac{x-1}{x+1}$

$\therefore\ xX + X = x - 1$

$\therefore\ x = \frac{X+1}{1-X}$

Given $x^2 - 2x + 3 = 0$

$\therefore\ \left(\frac{x+1}{1-x}\right)^2 - 2\left(\frac{x+1}{1-x}\right) + 3 = 0$

$\Rightarrow (x+1)^2 - 2(x+1)(1-x) + 3(1-x)^2 = 0$

$\Rightarrow 6x^2 - 4x + 2 = 0$

16. $ax^2 + bx + c = 0$

$D = b^2 - 4ac$

Roots are imaginary when

D is – ive

i.e. $b = 0$ & $a > 0, c > 0$.

17. $\because$ Roots are reciprocal

$\therefore \alpha\beta = 1$

$\frac{-2}{\ell} = 1$

$\therefore \ell = -2$

18. If one root of $ax^2 + bx + c = 0$ tends to infinity then a = 0.

Here equation is

$x(x + 2) = 4 - (1 - ax^2)$

$\Rightarrow x^2 + 2x = 3 + ax^2$

$\Rightarrow (1 - a)x^2 + 2x - 3 = 0$

$\therefore 1 - a = 0$

$\therefore a = 1$

19. Given equation are

$(6k + 2)x^2 + rx + (3k - 1) = 0$

$(12k + 4)x^2 + px + (6k - 2) = 0$

$\because$ both roots are common

$\therefore \frac{6k+2}{12k+4} = \frac{r}{p} = \frac{3k-1}{6k-2}$

$\therefore \frac{2(3k+1)}{4(3k+1)} = \frac{r}{p} = \frac{3k-1}{6k-2}$

$\therefore \frac{1}{2} = \frac{r}{p}$

$\therefore 2r - p = 0$

20. Let the roots of $x^2 - 6x + a = 0$ are α, β

$\therefore \alpha + \beta = 6$ ...(i)

& $\alpha\beta = a$ ...(ii)

& let the roots of $x^2 - cx + 6 = 0$ are α, γ

$\therefore \alpha + \gamma = c$ ...(iii)

& $\alpha\gamma = 6$ ...(iv)

dividing (ii) by (iv) or $\frac{\beta}{\gamma} = \frac{a}{6}$

$\Rightarrow \frac{a}{6} = \frac{4}{3} \Rightarrow \boxed{a = 8}$

$\therefore$ First equation is $x^2 - 6x + 8 = 0$

$\Rightarrow (x - 2)(x - 4) = 0$

$\therefore x = 2$ or $x = 4$

*If common root $\alpha = 2$; then $\beta = 4$; $\gamma = 3$

*If common root $\alpha = 4$; then $\beta = 2$; $\gamma = 3/2$

21. $x^2 + 2x + 3 = 0$ ....(1)

$D = 4 - 4 \times 3 = -8 < 0$

roots are imaginary

$ax^2 + bx + c = 0, \quad a, b, c \in R$ ....(2)

Eq. (1) & (2) have one root common and roots of (1) are imaginary so both roots will be common

$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

a : b : c = 1 : 2 : 3

22. $\because \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\Rightarrow \frac{-b}{a} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$\Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$

$\Rightarrow \frac{-bc^2}{a} = b^2 - 2ac$

$\Rightarrow -bc^2 = ab^2 - 2a^2c$

$\therefore a^2c = \frac{ab^2 + bc^2}{2}$

$\therefore ab^2, ca^2, bc^2$ are in AP.

23. $3x^2 + ax + 1 = 0$

$2x^2 + bx + 1 = 0$

have a common root

$\therefore \begin{vmatrix} a & 1 \\ b & 1 \end{vmatrix} \cdot \begin{vmatrix} 3 & a \\ 2 & b \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}^2$

$\Rightarrow (a - b)(3b - 2a) = 1$

$\Rightarrow -2a^2 + 5ab - 3b^2 = 1$

24. $y = \frac{2x^2 + 4x + 1}{x^2 + 4x + 2}$

$\Rightarrow (2 - y)x^2 + (4 - 4y)x + (1 - 2y) = 0$

$\because x \in R \rightarrow \therefore D \geq 0$

$\Rightarrow 16(1 - y)^2 - 4(1 - 2y)(2 - y) \geq 0$

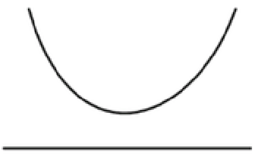
$\Rightarrow (4 + 4y^2 - 8y) - [2 - 5y + 2y^2] \geq 0$

$\Rightarrow 2y^2 - 3y + 2 \geq 0$

Here a = 2 > 0

& $D = 9 - 4.2.2 = -7 < 0$

So



$\therefore$ $2y^2 - 3y + 2 > 0$ for all real values of y.

25. $7^{\log_7(x^2-4x+5)}$

$= x - 1$

is valid when $\boxed{x^2 - 4x + 5 > 0}$

Now $x^2 - 4x + 5 = x - 1$

$\Rightarrow$ $x^2 - 5x + 6 = 0$

$\Rightarrow$ $x = 2, 3$

$x = 2$ $\Rightarrow$ $x^2 - 4x + 5 = 1 > 0$

$x = 3$ $\Rightarrow$ $x^2 - 4x + 5 = 2 > 0$

26. Clearly roots are

$3\alpha + 2,\ 3\beta + 2$

27. $\because$ $y = 0$

$\therefore$ $x^2 + ax + 25 = 0$

where D = 0 (coincident roots)

$\therefore$ $a^2 = 100$

$\Rightarrow$ $a = \pm 10$

28. We have, $2a^2x^2 - 2abx + b^2 = 0$

Discriminant, $D = (-2ab)^2 - 4(2a^2)(b^2)$

$= 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$

Roots are always complex.

29. $\because$ $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

Also, $\alpha + h + \beta + h = -\dfrac{q}{p}$

$\Rightarrow$ $\alpha + \beta + 2h = -\dfrac{q}{p}$

$\Rightarrow$ $2h = -\dfrac{q}{p} + \dfrac{b}{a}$ $\left(\because \alpha + \beta = -\dfrac{b}{a}\right)$

$\Rightarrow$ $h = \dfrac{1}{2}\left[\dfrac{b}{a} - \dfrac{q}{p}\right]$

30. $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$

Let $x^2 + 2 = y$

$y^2 + 8x^2 = 6xy$

$y^2 - 6xy + 8x^2 = 0$

$y = \dfrac{6x \pm \sqrt{36x^2 - 32x^2}}{2}$

$y = \dfrac{6x \pm 2x}{2} = 3x \pm x$

$y = 4x, 2x$

At $y = 4x$,

$x^2 + 2 = 4x$

$x^2 - 4x + 2 = 0$

Discriminant, $D = 16 - 8 = 8 > 0$

Roots are real.

Sum of roots $= -(-4) = 4$

At $y = 2x$,

$x^2 + 2 = 2x$

$x^2 - 2x + 2 = 0$

$D = 4 - 8 = -4 < 0$

Roots are complex.

Sum of roots = 2

Sum of all roots = 4 + 2 = 6

only statement 2 is correct.

$\therefore$ Correct option is (b)